

MEMORANDUM ON THE CONSTRUCTION OF LIFE TABLES Nos. 4 AND 4A, 1945-47.

DATA USED IN THE CONSTRUCTION OF THE LIFE TABLES.

Life tables appertaining to two areas were prepared, these areas being Ireland (exclusive of the Six Counties) and the aggregate of the four County Boroughs, Dun Laoghaire Borough and the Urban Districts of the State.

For ages 5 years and over the construction of the life tables was based on the Census Population, 12th May, 1946, and the number of deaths in 1945, 1946 and 1947, obtained from the statistics collected by the Registrar-General. For the infantile ages the Registrar-General's statistics of births and deaths were used because the numbers of children at the very young ages are somewhat understated at the Census. The population in each age-group was not adjusted to the middle of the Census year, as is usually the practice in other countries, because in Ireland there is no reason to believe that the population on 30th June, 1946, is closer than the population on Census date to the average population of the three years 1945-47. The grouped population data utilised in the life tables for Ireland (exclusive of the Six Counties) are given in Table 16, and in respect of Urban Districts grouped population data were extracted from basic tables used in the compilation of Table 18 of this Volume. The Registrar-General's statistics of births and deaths in Ireland are appended. The corresponding statistics relating to Urban Districts are not published by the Registrar-General.

METHOD OF COMPUTATION.

Previous Irish Life Tables were computed by a method closely following that first used by Mr. George King* in the construction of English life tables. This method applied to Irish data involved the calculation of graduated pivotal values of the population and annual average deaths at certain ages, every fifth year being chosen from age 17 to 32 and every tenth year from age 40 to 80, the use of these decennial intervals being considered necessary on account of the misstatement of age at death after middle-age. Apart from the undesirability of changing from quinquennial to decennial intervals, the comparison of actual and expected deaths was unsatisfactory in the older age-groups. When this method of construction was applied to the 1946 data the comparison of actual and expected deaths was less satisfactory than on the previous occasions, and accordingly it was felt that a life table constructed in this manner was not a true reflection of mortality in the State. A number of different methods of construction were tested and the method described in the following paragraphs was finally adopted as being the best.

It was decided that accurate quinquennial pivotal values were preferable to the use of decennial pivotals, but preliminary computation had shown that there was a marked overstatement of deaths in the age-groups ending with the digits 0 to 4 and a marked understatement in the other age-groups at ages above 45 years. It was, therefore, considered essential to redistribute the deaths into 5-year age-groups, keeping the number of deaths in the age-groups 45-54, 55-64, etc., constant. To do this the number of expected deaths in the 1946 population in each quinquennial age-group above 45 years was calculated by using the 1941 life tables. The actual number of deaths in the group 45-54 years was divided *pro rata* with the expected numbers obtained in the groups 45-49 and 50-54 years, and the resulting figures were used for the numbers of deaths at ages 45-49 and 50-54 years. The numbers of deaths in the other quinquennial age-groups were found in a similar manner.

The population in the Urban Districts was available only in decennial groups at the later ages and it was necessary to divide these into quinquennial groups. This was effected by dividing each decennial group *pro rata* with the numbers of males or females in the Town Areas in the corresponding quinquennial groups. Use was also made of the Town Areas' population in conjunction with the 1941 Urban Life Table to effect the redistribution of the Urban deaths into quinquennial groups.

Graduated pivotal values of the population and deaths at quinquennial intervals from age 12 to age 97 years were then calculated, using King's formula $V_x = 0.216W_x - 0.008(W_{x-5} + W_{x+5})$. From these values pivotal values of m_x and hence q_x were computed. After examination the pivotal values of q_x for $x=97$ were rejected in the case of the life tables for the State and values of q_x for $x=87$, 92 and 97 were rejected in the Urban Life Table. The series of pivotal values was completed by calculating q_7 from mortality and population data in the age-groups 2-4, 5-9 and 10-14 years.

* Supplement to the 75th Annual Report of the Registrar-General of Births, Deaths and Marriages in England and Wales, pt. I, Life Tables (Col. 7512/1914).

Osculatory interpolation was used to calculate the values of q_x from $x=12$ to 87 in the life table for the State and from $x=12$ to 77 in the Urban life table. The function $\log(q_x+0.1)$ was used in the interpolation and the method was that adopted by King. To extend the values of q_x beyond these ages a second degree curve was fitted to the last three pivotal values of q_x retained. It was found that such a curve gave the most reasonable values of q_x at the old ages.

The values of q_x from $x=0$ to 4 were derived solely from the deaths in the three years 1945, 1946 and 1947 and from the births registered in each quarter from 1940 to 1947 in the case of the life table for the State and from the births registered in each year in the case of the Urban life table. In these calculations the formula devised by Sir Alfred Watson* for English Life Table No. 9 was used.

The remaining values of q_x from $x=5$ to $x=11$ were obtained by Langrangean interpolation through the points $x=3, 4, 7, 12$ and 13.

Although seven decimal places were retained in the calculation of all q_x 's, the values of q_x were subsequently rounded off to five decimal places and these values used in the computation of the remaining columns of the life tables. In computing the l_x and d_x columns each d_x was rounded off to a whole number before subtracting from the corresponding \bar{l}_x to obtain l_x+1 . The tables were ended at the value $x=105$. Values of L_x (except L_0) were obtained from the relation $L_x = \frac{1}{2}(l_x + l_{x+1})$. L_0 was found by adding to L_1 the number of years of life lived by those who died before reaching age 1, which number was based on the deaths at ages under 1 month, 1-2 months, 2-3 months, 3-6 months and 6-12 months. Values of T_x were obtained by accumulating the values of L_x from the oldest age in the table down to age 0. Values of e_x were obtained directly from the published l_x and T_x columns except at the old ages where the number of deaths each year was below 1,000. At these ages, in order to keep the function e_x of smooth graduation right to the end of the table, four significant figures or four decimal places (whichever amounted to the lesser number of figures) were retained in each d_x and the l_x , L_x and T_x columns were computed accordingly. The values of e_x obtained by this method were inserted in the published tables.

THE ACTUAL-EXPECTED TEST.

In order to show that life tables accurately reflect mortality conditions it is customary to compare in each age-group the actual number of deaths with the "expected" number of deaths which represents the group sum product of the Census population and the life table death-rate at each year of life. At the later ages this test can be applied only to decennial groups on account of the misstatement of age and the redistribution of the deaths into quinquennial age-groups. In the following table the actual and expected numbers of deaths in Ireland are compared for ages 5 and over.

AGES	MALES			FEMALES		
	Deaths 1945-47 Annual Average		Deviation : Expected less Actual	Deaths 1945-47 Annual Average		Deviation : Expected less Actual
	Actual	Expected		Actual	Expected	
5-9 ...	184	178	- 6	171	170	- 1
10-14 ...	148	154	+ 6	145	145	0
15-19 ...	292	288	- 4	310	307	- 3
20-24 ...	380	379	- 1	421	418	- 3
25-29 ...	360	360	0	401	403	+ 2
30-34 ...	373	375	+ 2	417	416	- 1
35-39 ...	425	424	- 1	425	428	+ 3
40-44 ...	488	488	0	465	461	- 4
45-54 ...	1,539	1,532	- 7	1,349	1,346	- 3
55-64 ...	2,844	2,847	+ 3	2,844	2,854	+10
65-74 ...	5,847	5,859	+12	4,858	4,868	+10
75-84 ...	5,233	5,192	-41	4,929	4,883	-46
85-94 ...	1,189	1,190	+ 1	1,421	1,414	- 7
95+ ...	78	130	+52	113	144	+31
TOTAL 5 Years and over	19,380	19,396	+16	17,769	17,757	-12

* The Registrar-General's Decennial Supplement, England and Wales, 1921. Part I.

It will be observed that at all ages except those over 95 years the correspondence is very satisfactory and that the accumulated deviations for all ages are very small. To some extent this satisfactory correspondence was obtained at the expense of smoothness in the rates of mortality at the later ages, where the third and fourth differences of q_x are somewhat large. It was thought preferable, however, to adhere to a good expected test rather than to extreme smoothness in the values of q_x .

THE RESULTS.

In the life table for the State q_x declines from birth to age 11 for males and to age 10 for females. Thereafter it increases constantly with age for males but in the case of females there is a slight recession from age 24 to 26. The mortality rates for females are higher than those for males at ages 4-6 and 12-41 inclusive but at all other ages they are lower.

In the Urban life table q_x declines from birth to age 9 for both sexes. Thereafter it increases constantly with age for males but in the case of females there is a slight recession from age 28 to 30. The mortality rates for females are higher than those for males at ages 13-32. At all other ages the female rates are lower.

The mortality of males is lower in the Urban Districts than in the State at ages 8-10 and 76-83, but at all other ages the rates in the Urban life tables are higher.

Females have a lower mortality rate in the Urban Districts at ages 6-9, 20-34 and 75-86 but at all other ages the Urban rates are higher.

The mortality rates and expectations of life at ages 0, 10, etc., to 80 in 1925-27, 1935-37, 1940-42 and 1945-47 are compared in the following tables:—

RATES OF MORTALITY, q_x — IRELAND*

Age x years	MALES				FEMALES			
	Life Table				Life Table			
	No. 1 1925-27	No. 2 1935-37	No. 3 1940-42	No. 4 1945-47	No. 1 1925-27	No. 2 1935-37	No. 3 1940-42	No. 4 1945-47
0	.07716	.07952	.08147	.07534	.06346	.06327	.06387	.06096
10	.00166	.00134	.00115	.00103	.00195	.00185	.00125	.00085
20	.00401	.00349	.00349	.00293	.00447	.00369	.00375	.00336
30	.00529	.00438	.00425	.00348	.00590	.00516	.00448	.00400
40	.00705	.00621	.00560	.00506	.00747	.00630	.00576	.00515
50	.01130	.01175	.01054	.00999	.01139	.01053	.00997	.00921
60	.02428	.02407	.02341	.02251	.02394	.02238	.02162	.01868
70	.04913	.05510	.05257	.05397	.04647	.05090	.04732	.04555
80	.11389	.10632	.12352	.13481	.10049	.09657	.10912	.11662

* Exclusive of the Six Counties.

EXPECTATION OF LIFE (YEARS), e_x — IRELAND*

Age x years	MALES				FEMALES			
	Life Table				Life Table			
	No. 1 1925-27	No. 2 1935-37	No. 3 1940-42	No. 4 1945-47	No. 1 1925-27	No. 2 1935-37	No. 3 1940-42	No. 4 1945-47
0	57.37	58.20	59.00	60.47	57.93	59.62	61.02	62.43
10	55.20	55.75	56.25	56.89	54.92	56.10	56.94	57.90
20	46.40	46.83	47.24	47.77	46.36	47.26	48.04	48.84
30	38.39	38.53	38.92	39.22	38.60	39.22	39.89	40.53
40	30.43	30.26	30.58	30.60	30.83	31.18	31.63	32.10
50	22.67	22.41	22.53	22.43	23.19	23.28	23.54	23.88
60	15.75	15.46	15.37	15.12	16.36	16.17	16.31	16.41
70	10.02	9.99	9.60	9.21	10.72	10.59	10.42	10.19
80	5.81	6.00	5.66	5.27	6.47	6.46	6.39	5.99

* Exclusive of the Six Counties.

RATES OF MORTALITY, q_x —URBAN AREAS

Age x years	MALES		FEMALES	
	Life Table			
	No. 3A 1940-42	No. 4A 1945-47	No. 3A 1940-42	No. 4A 1945-47
0	.10499	.09294	.08312	.07730
10	.00116	.00098	.00140	.00086
20	.00410	.00317	.00368	.00326
30	.00437	.00363	.00421	.00375
40	.00706	.00598	.00592	.00544
50	.01566	.01430	.01134	.01014
60	.03374	.03272	.02524	.02103
70	.06703	.06551	.05516	.05227
80	.12795	.12881	.10858	.11028

EXPECTATION OF LIFE (YEARS) e_x —URBAN AREAS

Age x years	MALES		FEMALES	
	Life Table			
	No. 3A 1940-42	No. 4A 1945-47	No. 3A 1940-42	No. 4A 1945-47
0	54.88	56.77	58.43	60.46
10	53.02	54.20	55.76	57.10
20	44.11	45.16	46.96	48.14
30	35.83	36.57	38.64	39.71
40	27.49	28.03	30.28	31.23
50	19.92	20.22	22.26	23.09
60	13.60	13.75	15.34	15.85
70	8.80	8.88	9.94	10.09
80	5.18	5.26	6.02	6.10

The marked decline in the mortality rate at the younger ages (except age 0) since 1925-27 will be noted. The increases recorded at the older ages are probably due to an improvement in statement of age at death. Between 1940-42 and 1945-47 there was an improvement in the expectation of life at all ages up to 41 for males and up to 62 for females in the State as a whole. An improvement was recorded in the Urban Districts at all ages for both males and females.