MEMORANDUM

ON THE CONSTRUCTION OF

SAORSTAT LIFE TABLE NO. 1.

The methods used by Mr. George King in the construction of English Life Tables Nos. 7 (1901–10) and 8 (1910–12)* and adopted by Sir Alfred Watson in the construction of English Life Table No. 9 (1920 22)† were used in the construction of the table (pp. 216–219). Slight modifications in the methods were rendered necessary by the nature of the data available here.

At ages 5 and over the life table data were calculated from the population in 1926 as recorded at the Census and the numbers of deaths in 1925, 1926 and 1927 as recorded in the annual returns of the Registrar-General of Saorstát Eireann. From these statistics the "central death rate" m_x which equals

$\frac{1}{3}$ (deaths at age x in the years 1925, 1926 and 1927)
$\div$ population at age x at the 1926 Census

was calculated at certain "pivotal" ages x.

As mortality data by single years of age were not available and as the accurate statement of age at the Census left much to be desired, it was necessary to calculate "graduated" numbers of deaths and population at the pivotal ages from mortality and Census statistics suitably grouped. The method by which these graduated functions were derived from the grouped data is described by Mr. King. Both mortality and population statistics were available by single years in England and Wales in the years 1910–12 and Mr. King found that most accurate results would be obtained by using the quinquennial grouping 4–8, 9–13, etc., to 99–103. (Sir Alfred Watson used the grouping 2–6, 7–11, 12–16, etc.). Then from deaths at ages 4–8, 9–13 and 14–18 a graduated number of deaths at the central age 11 was calculated and similarly a graduated population at age 11. The quotient gave the central death rate m_{11} at age 11. Similarly from the age groups 9–13, 14–18 and 19–23 the central death rate at age 16 was found, and so on. From these values of m_x at ages 11, 16, 21, etc., values of q_x, the probability at exact age x of dying during the ensuing year was found from the formula $q_x = \frac{2m_x}{2+m_x}$.

Most of the intervening values of q_x were calculated by the method of "osculatory interpolation," also described by Mr. King, the values of q_{11}, q_{16}, q_{21}, and q_{26} being used to calculate the values of q_{17}, q_{18}, q_{19} and q_{20}, the values of q_{16}, q_{21}, q_{26} and q_{31} being used to calculate the values between q_{21} and q_{26}, etc. At the ages 0 to 5 the values of q_x were based solely on vital statistics, and at the old ages on the values of q_x at ages 88, 89, 90, 91 and 96.

These methods had to be modified for application to Saorstát statistics. In the first place, deaths at ages 5 and over were available only by quinquennial age groups 5–9, 10–14, etc. Choice of grouping was therefore restricted to these groups or to combinations of them. In the second place, the very marked tendency at the Saorstát Census towards giving ages in round numbers 40, 50, 60, etc., rendered it necessary to adopt a wider age grouping than that used by Mr. King. It was decided to use quinquennial grouping from ages 5–9 to 40–44 and (as these inaccuracies of age statement were more pronounced at the older ages) a partially overlapping decennial grouping from 25–34 to 95–104. From the quinquennial groups graduated numbers of deaths and of populations (and hence the values of q_x) at ages 12, 17, etc., to 37 were obtained and from the decennial grouping the values of q_x at ages 40, 50, etc., to 90. All values of q_x from $x=17$ to $x=32$ were calculated by osculatory interpolation from the quinquennial data [using, with Mr. King, the function $log\ (q_x+.1)$] and from the values of q_{30}, q_{40}, etc., to q_{90} the values of q_x for single years from $x=40$ to $x=80$ were obtained. The values from $x=33$ to $x=39$ were calculated by a Lagrangean interpolation from the values of q_{31}, q_{32}, q_{40}, and q_{41}. Thus unbroken series of values of q_x for males and females separately were found from $x=17$ to $x=80$.

* Supplement to the 75th Annual Report of the Registrar-General of Births, Deaths and Marriages in England and Wales, Part I. Life Tables (Cd. 7512/1914).

† The Registrar-General's Decennial Supplement, England and Wales, 1921. Part I. Life Tables (1927).

The values of q_x for ages 0 to 4 were derived solely from births and from deaths at ages 0, 1, 2, 3 and 4 using Sir Alfred Watson's formulae‡. From the mortality and population data grouped in ages 2–4, 5–9 and 10–14 the graduated number of deaths and of population at age 7 (and hence the value of q_7) were calculated. The remaining values of q_x from $x=5$ to $x=16$ were calculated by a Lagrangean interpolation from the known values of q_3, q_4, q_7, q_{12}, q_{17}, and q_{18}.

Mr. King obtained the values of q_x for advanced ages from a fourth difference formula based on the values of q_{88}, q_{89}, q_{90}, q_{91}, and q_{96}. Sir Alfred Watson used a "Gompertz" graduation to obtain the values of q_{85} and upwards as it was observed that the ratio $\frac{log\ p_{89}}{log\ p_{84}}$ $(p_x = 1 - q_x)$ was approximately equal to $\frac{log\ p_{94}}{log\ p_{89}}$. This condition was tested for the Saorstát pivotal values q_{70}, q_{80}, and q_{90}, and as it did not apply with sufficient accuracy a "Makeham" graduation based on the values of p_{70}, p_{80} and p_{90} was used instead.

The resulting series of values of q_x from $x=0$ to $x=107$ are of smooth graduation. That they reflect mortality in Saorstát Eireann accurately will be seen from the following table in which the actual deaths in the three years 1925–1927 are compared with the deaths as calculated from the life table :

Ages	Males			Females		
	Actual Deaths 1925–1927	Expected Deaths 1925–1927	Deviation: Expected less Actual	Actual Deaths 1925–1927	Expected Deaths 1925–1927	Deviation: Expected less Actual
0–4	10,633	10,633	—	8,900	8,900	—
5–9	1,080	1,062	—18	1,132	1,111	—21
10–14	751	760	+ 9	900	908	+ 8
15–19	1,413	1,371	—42	1,572	1,535	—37
20–24	1,640	1,642	+ 2	1,687	1,693	+ 6
25–34	3,139	3,151	+12	3,533	3,535	+ 2
35–44	3,647	3,634	—13	3,822	3,806	—16
45–54	5,810	5,867	+57	5,392	5,460	+68
55–64	8,579	8,579	—	7,717	7,706	—11
65–74	12,944	13,010	+66	13,086	13,050	—36
75–84	10,822	10,860	+38	11,817	11,809	— 8
85–94	3,908	3,981	+73	4,236	4,299	+63
95 and over	408	449	+41	499	504	+ 5
Total 5 years and and over	54,141	54,366	+225	55,393	55,416	+23

As the values of q_x from $x=0$ to $x=4$ were calculated from vital statistics only, actual and expected deaths have been entered at the same figures in the foregoing table. It will be seen that in each of the remaining age groups there is a satisfactory correspondence between actual and expected deaths. In the aggregate of all deaths of males 5 years of age and over the discrepancy is only +225 in 54,141 actual deaths. The discrepancy is only +23 in 55,393 actual deaths of females 5 years and over.

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‡ *Op. cit. p.* 30.